Periodic long-term X-ray and radio variability of Cygnus X-1

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ABSTRACT
We present a comprehensive analysis of long-term periodic variability of Cyg X-1 using the method of multiharmonic analysis of variance applied to available monitoring data since 1969, in X-rays from Vela 5B, Ariel 5, Ginga, CGRO and RXTE satellites and in radio from the Ryle and Green Bank telescopes. We confirm a number of previously obtained results, and, for the first time, find an orbital modulation at 15 GHz in the soft state and show the detailed non-sinusoidal shape of that modulation in the hard state of both the 15-GHz emission and the X-rays from the RXTE/All-Sky Monitors (ASM). We find the CGRO/BATSE data are consistent with the presence of a weak orbital modulation, in agreement with its theoretical modelling as due to Compton scattering in the companion wind. We then confirm the presence of a ∼150-d superorbital period in all of the data since ∼1976, finding it in particular for the first time in the Ariel 5 data. Those data sets, covering >65 superorbital cycles, show a remarkable constancy of both the period and the phase. On the other hand, we confirm the presence of a ∼290-d periodicity in the 1969–1979 Vela 5B data, indicating a switch from that period to its first harmonic at some time ≲1980. We find the superorbital modulation is compatible with accretion disc precession. Finally, we find a significant modulation in the RXTE/ASM data at a period of 5.82 d, which corresponds to the beat between the orbital and superorbital modulations provided the latter is prograde.

Key words: accretion, accretion discs – binaries: general – stars: individual: Cyg X-1 – radio continuum: stars – X-rays: binaries – X-rays: stars.

1 INTRODUCTION
Cyg X-1, a persistent Galactic black hole binary, was discovered in 1964 June (Bowyer et al. 1965), and it has been extensively studied since then. Its orbital period is \( P_{\text{orb}} = 5.6 \text{ d} \) and the optical component, HDE 226868/V1357 Cygni (Bolton 1972; Webster & Murdin 1972) is an OB supergiant (Walborn 1973). The masses of the two stars remain the subject of some dispute, with the most recent determination of 40 ± 10 and 20 ± 5 \( M_\odot \) for the supergiant and the black hole, respectively (Ziolkowski 2005).

Focused wind is the main accretion channel, though the companion nearly fills its Roche lobe (Gies & Bolton 1986; Gies et al. 2003, hereafter G03). Bound–free absorption by the wind leads to a strong modulation with the orbital period of the X-rays (Priedhorsky, Brandt & Lund 1995; Zhang, Robinson & Cui 1996; Brocksopp et al. 1999a, hereafter B99; Wen et al. 1999, hereafter W99; Kitamoto et al. 2000, hereafter K00) although the modulation is relatively irregular, manifesting itself by dips in the X-ray light curve near the superior conjunction of the black hole (Balucinska-Church et al. 2000). The modulation is also seen in the radio (B99; Pooley, Fender & Brocksopp 1999, hereafter P99), being then due to free–free absorption by the wind (Brocksopp, Fender & Pooley 2002). The optical emission is also modulated with the orbital period, showing two minima/maxima per period due to the ellipsoidal shape of the star (Lyutyi 1985; Kemp et al. 1987; Voloshina, Lyutyi & Tarasov 1997; Brocksopp et al. 1999b).

Typical of X-ray binaries, Cyg X-1 shows two main spectral states, hard and soft (e.g. Gierliński et al. 1997, 1999; Frontera et al. 2001; McConnell et al. 2002; Zdziarski & Gierliński 2004). Interestingly, the orbital modulation disappears in the soft state (W99), probably due to strong photoionization of the wind by the enhanced soft X-ray flux.

Cyg X-1 was also found to display superorbital modulation, i.e. at much longer time-scales than \( P_{\text{orb}} \). Originally, a period of 294 d was...
reported by Priedhorsky, Terrell & Holt (1983, hereafter PTH83) in X-rays and by Kemp et al. (1983) in the optical. Later, however, a \( \sim 150 \)-d period from the radio to X-rays was found by numerous authors (B99; P99; K00; Benlloch et al. 2001; Karitskaya et al. 2001; Ozdemir & Demircan 2001; Benlloch et al. 2004). The probable cause of this modulation is precession of the accretion disc.

We present here a comprehensive analysis of the orbital and superorbital modulation of Cyg X-1 in the X-ray and radio bands. We use most of the available X-ray and radio monitoring data, applying to it the method of multiharmonic analysis of variance (hereafter abbreviated as mhAoV). Section 2 presents details of our data selection, and Section 3, our analysis method. Sections 4 and 5 present our results on the orbital and superorbital modulation, respectively. Discussion and theoretical interpretation are given in Section 6, and our conclusions, in Section 7. In Appendix A, we separately reanalyse the presence of the \( \sim 290 \)-d periodicity in the Vela SB data. In Appendix B, we use the method of structure–function (SF) as an independent check of our results.

## 2 DATA SELECTION

We analyse available light curves separately for the hard and soft states. We follow the definitions of those states of Zdziarski et al. (2002); thus the state boundaries may slightly change if other definitions are used. We use X-ray data from the Burst and Transient Source Experiment aboard Compton Gamma Ray Observatory (CGRO/BATSE; Paciesas et al. 1997), and the All-Sky Monitors (ASM) aboard Rossi X-ray Timing Explorer (RXTE; Bradt, Rothschild & Swank 1993; Levine et al. 1996), Ginga (Makino et al. 1987; Tsunemi et al. 1989), Ariel 5 (Holt 1976) and Vela-SB (Conner, Evans & Belian 1969). We use the 15-GHz radio data from the Ryle Telescope of the Mullard Radio Astronomy Observatory, and the 2.25- and 8.30-GHz data from the Green Bank Interferometer (GBI) of the National Radio Astronomy Observatory in Green Bank, WV, USA. Table 1 gives the log of the data, which are presented graphically in Fig. 1.

For the RXTE/ASM, we use the dwell count rates from the Definitive Products data base of the High Energy Astrophysics Science Archive Research Centre (HEASARC). Each dwell lasts \( \sim 90 \) s (Levine et al. 1996) and its number within 1 d varies up to 30 for Cyg X-1. For the hard state, we have excluded high-flux intervals dominated by the so-called failed state transitions (Potschmidt et al. 2003). In this selection procedure, we have chosen MJD 50660–50990 as the reference interval, during which the count rate in each of the three energy bands (1.5–3, 3–5 and 5–12 keV) remained within 4\( \sigma \) around the respective mean. Then, we have searched 30-d intervals in the remaining parts of the hard state, and have excluded the intervals with more than 40 per cent of points in that interval exceeding the 4\( \sigma \) level. This, with some further minor adjustments, resulted in the intervals of MJD 50590–50660, 50995–51025, 51440–51640 and 51840–51960 excluded from the first part of the hard state of Table 1. We have furthermore omitted negative count rates, which comprise \( < 0.04 \) of the ASM data points, and thus their omission has a negligible effect. We have also applied the barycentre correction to the ASM times of measurements; however, its effect is negligible. We note that the behaviour of Cyg X-1 after MJD 52853 until now (\( \sim 53600 \)) has not been suitable for our analysis as showing alternate periods of the hard and soft states of rather short durations.

We notice that there is no agreement in the literature on the definition of the soft state. For example, Belloni et al. (1996) called the 1996 soft state of Cyg X-1 ‘intermediate’ whereas numerous other authors termed it ‘soft’. In particular, the two occurrences of the soft state considered by us (see Table 1), while characterized by the relatively high and steady high X-ray flux (see Fig. 1a), may be classified as containing also the intermediate state based on some other criteria. In general, blackbody–disc dominated states of black hole binaries range between ultrasoft, with almost no high-energy tail, to very high or intermediate, where the high-energy tail starts at the top of the disc–blackbody (see e.g. Zdziarz & Gierlinski 2004, for a review), with different timing properties of the disc–blackbody and the tail. The soft state of Cyg X-1 lies inbetween the ultrasoft and very high states, see e.g. the spectra in Gierlinski et al. (1999), Frontera et al. (2001) and Gierlinski & Zdziarski (2003).

We use the same BATSE data as Zdziarz et al. (2002). They contain 2729 d of usable observations between 1991 April 25 and 2000 May 22 in the energy bands of 20–100 keV and 100–300 keV. However, we exclude the periods of the soft state of 1994 and 1996, i.e. MJD 49250–49440 and MJD 50230–50307. Both of those occurrences of the soft state are of relatively short duration. For the former, only the BATSE data are available, and the latter has already been extensively studied, with W99 finding no orbital modulation in the RXTE/ASM data.

We use the Ariel 5/ASM 3–6 keV and Vela 5B/ASM 3–12 keV (averaged prior to analysis over 0.25-d intervals) light curves from the HEASARC data base. We note that we use the Ariel 5 data (Table 1) excluding the MJD 42338–42829, which were dominated by soft flare events and included the soft state of 1975 (Liang & Nolan 1984). We have also omitted data points with negative fluxes. We then use the same Ginga/ASM 1–20 keV data as K00. They contain 339 measurements during 1987 February–1991 October. The Ryle radio data used by us contain 10-min average flux measurements. Parts of this data set have been studied by P99 and Benlloch et al. (2004). We have divided the data into the hard and soft states (Table 1) based on the RXTE/ASM data. Interestingly, we see a relatively high level of the 15-GHz emission during the soft states. The 2.25- and 8.30-GHz GBI data (with the average sampling interval of \( \sim 0.9 \) d; shown in Fig. 1 with \( 1 \sigma \) errors) correspond to the hard state only. They have been screened according to the standard procedure.1 An early part of the GBI data set has been studied by P99 and B99.

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1 ftp://ftp.gb.nrao.edu/pub/gghigo/gbidata/gdata/00README

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### Table 1. The log of the light curves used in this work. H and S refer to the hard and soft spectral state, respectively. The two subintervals of the hard state from the RXTE/ASM and Ryle instruments are treated jointly in the analysis. See Section 2 for the criteria of data selection and for some additionally rejected intervals.

<table>
<thead>
<tr>
<th>State</th>
<th>Instrument</th>
<th>Start (MJD)</th>
<th>End (MJD)</th>
<th>Time span (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>RXTE/ASM</td>
<td>50350</td>
<td>52099</td>
<td>1750</td>
</tr>
<tr>
<td></td>
<td>Ryle</td>
<td>52565</td>
<td>52770</td>
<td>206</td>
</tr>
<tr>
<td></td>
<td>GBI</td>
<td>50409</td>
<td>51823</td>
<td>1415</td>
</tr>
<tr>
<td>S</td>
<td>RXTE/ASM</td>
<td>48371</td>
<td>51686</td>
<td>3316</td>
</tr>
<tr>
<td></td>
<td>Ryle</td>
<td>46887</td>
<td>48531</td>
<td>1645</td>
</tr>
<tr>
<td></td>
<td>Ariel 5/ASM</td>
<td>42830</td>
<td>44929</td>
<td>1463</td>
</tr>
<tr>
<td></td>
<td>Vela 5B/ASM</td>
<td>40368</td>
<td>44042</td>
<td>3675</td>
</tr>
</tbody>
</table>

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Figure 1. (a) The RXTE/ASM 1-d average light curves corresponding to hard states and the soft states of 2001–2002 and 2003. The used boundaries of the soft states are marked by dotted lines. The shaded regions correspond to the data not taken into account in our analysis in order to provide approximately the same flux level. (b) The 1-d average 15-GHz light curve from the Ryle telescope. (c) The 2.25- and 8.30-GHz GBI data. (d) The CGRO/BATSE light curves. The shaded regions correspond to the 1994 and 1996 soft states, which are not included in the analysis. (e) The Ginga/ASM data. (f) The Ariel 5/ASM light curve. (g) The Vela 5B/ASM 0.25-d average light curve.

3 TIME ANALYSIS

In our analysis, we first determine the significance of the presence of a period in a light curve using the method of mhAoV (Section 3.1). We use the logarithm of either flux or count rate, \( G(t) \equiv \ln \mathcal{F}(t) \), similarly to the use of magnitudes in optical astronomy. We do not take into account measurement errors in the analysis.

For a formulation and derivation of the quantitative results of the mhAoV method, see Schwarzenberg-Czerny (1999) and references therein. Then, for an established period, we fold and average the light curve and fit it with a Fourier series, which enables us to find the detailed shape of a periodic modulation including its phase (Section 3.2).

3.1 The method of period detection

3.1.1 General formulation and model orthogonality

In general, we learn from experiments by fitting data, \( x \), with a model, \( x_{\text{fit}} \). The data contain \( n \) measurements, and the model, \( n_{\text{f}} \) free parameters. The consistency of the data with the model is measured by a function, \( \Theta \), called a statistic. A given model, \( x_{\text{fit}} \), using a given statistic (e.g. \( \chi^2 \)), yields its particular value, \( \Theta_1 \). Various methods used in the analysis of time series differ both in their choice of the model and the statistic; hence are difficult to compare directly. To enable such a comparison and for determining the significance of results, \( \Theta \) is converted into the false alarm probability, \( P_1 \). This is done considering a hypothetic situation, \( H_1 \), in which \( x \) is pure white.
noise. Then each pair \((x_i, \Theta)\) corresponds to certain cumulative probability distribution of \(\Theta\), namely \(P(n_i, n; \Theta)\), with \(P\) being the tail probability that under the hypothesis \(H_i\) the experiment yields \(\Theta > \Theta_i\), i.e. \(P_1(\Theta > \Theta_i) = 1 - P(n_i, n; \Theta_i)\).

Up to here, we have just outlined the classical Neyman–Pearson procedure of statistics. The specific method for analysis of time series used here differs from those commonly encountered in astronomy only in the choices of \(x_i\) and \(\Theta\). Then, our accounting for variance, correlation and multiple frequencies in calculating \(P\) is dictated by the laws of statistics. The probabilities derived by us from the data are the false alarm probabilities. However, we also call them below just probabilities or significance levels.

We note then that Fourier harmonics are not orthogonal in terms of the scalar product with weights at unevenly distributed observations. Certain statistical procedures employing classical probability distributions hold for orthogonal models only and fail in other cases. To avoid that, a popular variant of the power spectrum, Lomb (1976) and Scargle (1982) (hereafter LS) periodogram relies on a special choice of phase such that the sine and cosine functions become orthogonal. We extend this approach by employing Szegő orthogonal trigonometric polynomials as model functions. A series of \(n_1 = 2N + 1\) polynomials corresponds to the orthogonal combinations of the \(N\) lowest Fourier harmonics (Schwarzenberg-Czerny 1996). Orthogonal series are optimal from the statistical point of view because, by virtue of the Fisher lemma (Fisz 1963; Schwarzenberg-Czerny 1998), they guarantee the minimum variance of the fit residuals for a given model complexity (given by \(n_1\)). Szegő polynomials are also convenient in computations since the least-squares solution may be obtained using recurrence and orthogonal projections, resulting in high computational efficiency, with the number of steps \(\propto N\) instead of \(N^3\) for \(N\) harmonics.

### 3.1.2 Variance, the AoV statistics, and model complexity

The LS method employs the sine as a model, and the quadratic norm, \(\Theta_2 = \|x - x_i\|^2\), as the statistic. The corresponding probability distribution is \(\chi^2\) with two degrees of freedom. Prior to use of the \(\chi^2\) distribution, \(\Theta_2\) has to be divided by the signal variance, \(V\). However, \(V\) is usually not known and has to be estimated from the data themselves. Then, neither \(\Theta_2\) nor variance estimates are independent nor their ratio follows the \(\chi^2\) distribution, which effect has to be accounted for. A simple way to do it is to apply the Fisher AoV statistic, \(\Theta = (n - n_1)\|x_1\|^2/(n_1\|x - x_1\|^2)\). Hence we call our method, involving Szegő polynomials model and the AoV statistics, the mhAoV (Schwarzenberg-Czerny 1996). The probability distribution is then the Fisher–Snedecor distribution, \(F\), rather than \(\chi^2\), and \(P_1 = 1 - F(n_1, n_2; \Theta)\) where \(n_2 = n - n_1\). For everything else fixed, replacing \(\chi^2\) with \(F\) for \(n = 100\) yields an increase of \(P_1(\chi^2) = 0.001\) to \(P_1(F) = 0.01\). Thus, accounting for the unknown variance yields the mhAoV detection less significant, but more trustworthy. In this work, \(n\) usually is larger, for which \(P_1(F)/P_1(\chi^2)\) reduces to several.

Apart from the choice of the statistic, our method for \(N = 1\) differs from the LS one in the average flux being subtracted in the latter (thus yielding \(n_1 = 2\)) whereas a constant term is fitted in the former (which can be often of significant advantage, see Foster 1995). If the periodic modulation in the data differs significantly from a sinusoid (e.g. due to dips, eclipses, etc.), then our \(N > 1\) models account for that more complex shape and perform considerably better than the LS one. For example, we show in Section 4.1 that a folded RXTE/ASM light curve is much better described by a model with \(N = 2\) than \(N = 1\) with the probability of that improvement being by chance of \(\sim 10^{-13}\).

#### 3.1.3 Multiple trials

Probability can be assigned to a period found in data according to one of two statistical hypotheses. Namely, (i) one knows in advance the trial frequency, \(\nu_0\) (from other data), and would like to check whether it is also present in a given data set or (ii) one searches a whole range, \(\Delta\nu\), of \(N_{\text{eff}}\) frequencies and finds the frequency, \(\nu\), corresponding to the most significant modulation. The two cases correspond to the probabilities \(P_1\) and \(P_{N_{\text{eff}}}\) to win in a lottery after 1 and \(N_{\text{eff}}\) trials, respectively, i.e. they represent the false alarm probabilities in single and multiple experiments, respectively. They are related by \(P_{N_{\text{eff}}} = 1 - (1 - P_1)^{N_{\text{eff}}}\). Note that the hypothesis (ii) and the probability \(P_{N_{\text{eff}}}\) must be always employed in order to claim any new frequency in the object under study. The hypothesis (i) is rarely used. However, since \(P_1 < P_{N_{\text{eff}}}\), it is the more sensitive one. For this reason, we advocate its use in the situations where the modulation frequency is already known, and we aim at checking for its manifestation in the same object but in a new band, new data set, etc. We stress that we do not use the hypothesis (i) to claim any new frequency.

In this work, we use \(P_1\) for cases with the orbital periodicity and the \(\sim 150\)-d superorbital period, found before in numerous studies, and use \(P_{N_{\text{eff}}}\) for all other periodicities. We calculate \(P_1\) and \(P_{N_{\text{eff}}}\) applying all due corrections (see below), and requiring stability of a period after removal of a modulation with another frequency (prewhitening).

An obstacle hampering use of the (ii) hypothesis is that no analytical method is known to calculate \(N_{\text{eff}}\). The number \(N_{\text{eff}}\) corresponds to independent trials, whereas values of periodograms at many frequencies are correlated because of the finite width of the peaks, \(\delta\nu\), and because of aliasing. As no analytical method is known to determine \(N_{\text{eff}}\), Monte Carlo simulations have been used (e.g. Paltani 2004). Here, we use a simple conservative estimate, \(N_{\text{eff}} = \min(\Delta\nu/\delta\nu, N_{\text{calc}}/n)\), where \(N_{\text{calc}}\) is the number of the values at which the periodogram is calculated. The estimate is conservative in the sense that it corresponds to the upper limit on \(N_{\text{eff}}\), and thus the minimum significance of detection. This effect applies to all methods of period search (Horne & Baliunas 1986). In general, it may reduce significance of a new frequency detection for large \(N_{\text{eff}}\) as \(P_{N_{\text{eff}}} \gg P_1\). In practice, it underscores the role of any prior knowledge, in a way similar to the Bayesian statistics: with any prior knowledge of the given frequency we are able to use the hypothesis (i) to claim the detection with large significance (small \(P_1\)).

#### 3.1.4 Correlation length

The \(P_1\), and other common probability distributions used to set the detection criteria, are derived under the assumption of the noise being statistically independent. Often this is not the case, as seen, e.g. in light curves of cataclysmic variables (CVs). The correlated noise, often termed red noise, obeys different probability distribution than the standard \(P_1\), and hence may have a profound effect. For example, noise with a Gaussian autocorrelation function (ACF) correlated over a time interval, \(\delta\tau\), yields a power spectrum with the Gaussian shape centred at \(v = 0\) and the width \(\delta\nu = 1/\delta\tau\). It may be demonstrated that the net effect of the correlation on \(P_1\) in analysis of low frequency processes is to decimate the number of independent observations by a factor \(n_{\text{corr}}\), the average number of observations

\[ n_{\text{corr}} = \frac{\text{observations}}{\text{independent} \text{ events}}. \]
in the correlation interval \(\delta t\) (Schwarzenberg-Czerny 1991). Effectively, one should use \(n_{\perp}/n_{\text{corr}}\) and \(\Theta/n_{\text{corr}}\) instead of \(n_{\perp}\) and \(\Theta\) in calculating \(P_1\). This result holds generally, for both least squares and maximum likelihood analyses of time series. In the respect of the red noise, accretion-powered X-ray sources resemble CVs and for them the red noise can also have profound consequences, see e.g. the simulations of Kong et al. (2002). In order to take into account this effect here, we check the ACF of the fit residuals.

For independent observations, \(m = 2\) consecutive residuals have the same sign on average (e.g. Fisz 1963). Thus, counting the average length, \(m\), of series of residuals of the same sign provides an estimate of the number of consecutive observations being correlated, \(n_{\text{corr}}\). Note that \(m = n/l\) where \(l\) is the number of such series (both positive and negative). For correlated observations, the average length of series with the same sign is \(m = 2n_{\text{corr}}\), which allows us to calculate \(n_{\text{corr}}\).

Let \(\Theta\) denote the Fisher–Snedecor statistics from the mhAoV periodogram (i.e. from Fourier series fit) computed for \(n_{\perp} = 2N + 1\) parameters, \(n\) observations and \(n_{\perp} = n - n_{\perp}\) degrees of freedom. To account for \(n_{\text{corr}}\), we calculate \(P_1\) as follows,

\[
P_1 = 1 - F\left(n_{\perp}, n_{\perp}/n_{\text{corr}}, \Theta/n_{\text{corr}}\right) = I_1\left(n_{\perp}, n_{\perp}/n_{\text{corr}}, 2\right),
\]

where

\[
z = n_{\perp}/n_{\perp}/n_{\text{corr}},
\]

and \(I_1(a, b)\) is the incomplete (regularized) beta function (Abramovitz & Stegun 1971), see Schwarzenberg-Czerny 1998 and references therein. [In the popular application, MATHEMATICA (Wolfram 1996), that function is called BetaRegularized.]

### 3.2 The method of fitting folded light curves

After establishing the existence of a period, we characterize the shape of the modulation by fitting the light curve, which we fold and average using a binning appropriate for the given statistics and the complexity of its shape. The shown errors on the folded and averaged light curves are based on the dispersion (rms) of the fluxes pre-averaged over each real-time bin falling into a given phase bin. (The pre-averaging is done to avoid variability over time-scales shorter than the time length of a phase bin is unimportant for the analysis.) In fitting the light curves, we take into account the errors obtained in this way. All the uncertainties shown below are \(1\sigma\).

We assume the modulation corresponds to a multiplicative factor (rather than to an additive flux component). This is suitable to describe modulation due to phase-dependent absorption, but it also can describe other effects. Thus,

\[
F(t) = F_{\text{int}}(t)A_{\text{mod}}(t),
\]

where \(F_{\text{int}}(t)\) is the intrinsic flux and \(A_{\text{mod}}(t)\) is the modulation factor.

Our use of the flux logarithm, \(G(t) = \ln F(t)\), allows us then to describe the modulation as additive. Then we use non-linear least-squares method to obtain the best-fitting \(N\)-harmonic Fourier series,

\[
G_{\text{mod}}(t) = G_0 - \sum_{k=1}^{N} G_k \cos[2\pi k(v - T_0) - \phi_k].
\]

where \(t\) is measured from some initial moment, \(T_0\), \(G_0\) is a (fitted) constant term, and we use the same value of \(N\) as for the corresponding periodogram. The value of \(N\) is established by the \(F\)-test based on the values of \(\chi^2\) of the fit with equation (4). Note that we do not treat \(v\) as a free parameter but instead use either the values obtained from the periodograms or from other data (e.g. the orbital frequency from spectroscopy), compatible with fitting here the folded and averaged light curve.

The obtained modulation can be removed from the light curve by,

\[
G_{\text{int}}(t) = G(t) - G_{\text{mod}}(t) + G_0
\]

\[
= G(t) + \sum_{k=1}^{N} G_k \cos[2\pi k(v - T_0) - \phi_k],
\]

which preserves the average flux. On the other hand, if the modulation is caused by absorption, one can correct the light curve for the variable part of the absorption,

\[
G_{\text{int}}(t) = G(t) - G_{\text{mod}}(t) + G_{\text{max}},
\]

where \(G_{\text{max}} = \max[G_{\text{mod}}(t)]\), which increases the average logarithm of the flux by \((G_{\text{max}} - G_0)\), see Tables 2 and 3.

We remove the orbital modulation prior to searching for longer periods (pre-whitening), using equation (5), in order to reduce the noise level. Then we use the periodogram to identify the frequency of the next most prominent modulation, and fit the Fourier series of equation (4). Then we pre-whiten it again, i.e. divide the data by the new modulation. We repeat the procedure until the peak amplitude becomes close to the noise level. To test the persistence and significance of the detected modulation, we then repeat the procedure using a half of the data. On the other hand, we remove the main superorbital modulation found by us (see Section 5 below) before calculating final results for the orbital modulation.

We define the relative modulation depth by

\[
\phi_{\text{mod}} = \frac{F_{\text{max}} - F_{\text{min}}}{F_{\text{max}}},
\]

where \(F_{\text{min}}\) and \(F_{\text{max}}\) are the minimum and maximum flux of the fitted model. This definition, though it differs from the standard peak-to-peak one, is suitable for modulation caused by absorption. For \(N = 1\), the modulation depth and its standard deviation are,

\[
\phi_{\text{mod}} = 1 - \exp(-2G_1), \quad \delta \phi_{\text{mod}} = 2\exp(-2G_1)\delta G_1,
\]

where \(\delta G_1\) is the standard error of \(G_1\). For \(N > 1\), \(\phi_{\text{mod}}\) is found numerically, and \(\delta \phi_{\text{mod}}\) by propagation of errors.

### 3.3 Spectral windows

The discrete spectral window (DSW),

\[
|w(v)|^2 = n^{-2}\left(\sum_{k=1}^{n} \cos 2\pi v t_k + \sum_{k=1}^{n} \sin 2\pi v t_k\right)^2,
\]

where \(t_k\) is the time of the \(k\)th observation, identifies features related to regularities of the light curve sampling (e.g. Deeming 1975). We have found that only the DSW of the RXTE/ASM contains interesting features. Namely, there are two strong peaks at 52.6 and 0.96 d and their harmonics, see Fig. 2. We have also calculated the DSW for three other bright X-ray sources, Crab, Cyg X-3 and GX 5–1, and found the same peaks.

The likely cause of the 52.6-d peak is precession of the RXTE satellite, with period in the range of 50–60 d (A. M. Levine private communication). The precession causes a modulation in the frequency of passing through the South Atlantic Anomaly, when the detectors are switched off. The ~1-d feature appears to be due to RXTE scheduling (Zdziarski et al. 2004; Farrell, O’Neill & Sood 2005). A peak of the DSW at a frequency \(v\) may give rise to aliases
of actual modulation at frequency offsets of $\pm v$. However, we have found no such features in the studied periodograms.

## 4 ORBITAL MODULATION

### 4.1 The hard state

Whenever we find a period of $\simeq 5.6 \pm 0.3$ d, it is compatible within the measurement errors (Table 2) with the spectroscopic orbital period. Given this lack of evidence of any difference between the period of the modulation and the actual orbital period, we use the best available ephemeris for all the data sets in fitting the modulation shape. We use the period, $P$, of Brocksopp et al. (1999b) and choose $T_0$ based on LaSalas et al. (1998), which gives the ephemeris of

$$\text{min}[\text{MJD}] = 50234.79(\pm 0.01) + 5.599829(\pm 0.00002)E,$$  \hspace{1cm} (10)

where $E$ is an integer and $T_0$ was chosen to be as close as possible to all of the intervals of our main data sets.

Following the method of Section 3, we have found that the orbital modulation has a complex shape for some data sets, for which the sinusoidal description ($N = 1$) is clearly insufficient. Using the $F$-test (e.g. Bevington & Robinson 1992) for the fits on the rebinned folded light curves, we have found that $N = 3$ harmonics of the Fourier series of equation (4) are required, in particular, for the RXTE/ASM data. In the case of the 1.5–3 keV band and for the folded and averaged light curve rebinned into 100 bins, the probability that the chance improvement of the fit from $N = 1$ to $N = 2$ is $\sim 10^{-13}$, and from $N = 2$ to $N = 3$, 0.004. Then, $N = 4$ is not required, with the probability of 0.67. Fig. 3 compares the results for $N = 1$ and 3. The shape of the orbital modulation can be described as relatively flat for the normalized phase $\sim 0.3–0.7$ and with a sharp dip around the phase 0/1.

Our results are given in Table 2 (but note that we give the values of $n$ only in Table 3) and Fig. 4. We treat jointly the hard-state subintervals of the RXTE/ASM and Ryle data (Table 1). The fractional modulation in the RXTE/ASM data decreases with the increasing energy, and is in good agreement with the results of W99.

The 15-GHz radio data, see Table 2 and Fig. 4(d), also require $N = 3$. Notably, the minimum of the radio modulation is significantly shifted with respect to the zero phase, by 0.153, or 0.86 d. The corresponding ephemeris (using the spectroscopic period) is,

$$\text{min}[\text{MJD}] = 50235.65(\pm 0.01) + 5.599829E.$$  \hspace{1cm} (11)

A similar, but smaller (0.11 or 0.67 d) offset was earlier reported by P99. The difference appears to be due to both the fit with $N = 1$ and a shorter light curve (2 yr) used by them. In particular, the fit with $N = 1$ for the current data set yields an offset of 0.14.

The fractional modulation depth of the radio emission decreases with the decreasing frequency, see Table 2 and Figs 4(e) and (f). The quality of the 8.3- and 2.25-GHz data is also lower than that of the 15-GHz data, and $N = 1$ is sufficient to describe the modulation. Interestingly, the offset of the minimum increases with the decreasing frequency, reaching $0.32 \pm 0.09$ (1.8 $\pm 0.5$ d) at 2.25 GHz. Since $N = 1$, the values of the offset at 8.3 and 2.25 GHz follow directly from Table 1. These results are similar to those of P99, who studied a part of those data up to 1998.

For the 20–100 keV BATSE data, we calculate the mhAoV periodogram with $N = 1$, which yields $\Theta = 3.7$, corresponding to the probability of $P_1 \simeq 0.18$, see Table 2. Although this is a very weak significance, the 20–100 keV modulation is probably real given our a priori knowledge of the orbital period and the presence of very significant modulation in softer X-rays. Our results agree with
Table 3. The most significant superorbital periodicities found in the hard state. \( N = 1 \) in all cases, and \( n \) gives the total number of points in the light curves. See Appendix A for discussion of the Vela 5B data. The values of \( \mathcal{P}, \Theta, n_{\text{corr}}, P_1 \) and \( P_{\text{Neff}} \) are found directly from the periodograms whereas the remaining quantities are obtained for the folded and averaged light curves with the ephemeris of equation (12) used for the first 11 rows. Since \( N = 1 \) throughout, \( G_{\text{max}} - G_0 = G_1 \). The units of \( \exp (G_0) \) are those shown in Fig. 1.

<table>
<thead>
<tr>
<th>Detector</th>
<th>( \mathcal{P} ) [d]</th>
<th>( \Theta )</th>
<th>( n )</th>
<th>( n_{\text{corr}} )</th>
<th>( P_1 )</th>
<th>( \exp (G_0) )</th>
<th>( G_0 )</th>
<th>( G_1 )</th>
<th>( \varphi_1 )</th>
<th>( \varphi_{\text{mod}} ) [per cent]</th>
</tr>
</thead>
<tbody>
<tr>
<td>RXTE (1.5–3 keV)</td>
<td>152.60 ± 1.20</td>
<td>722.7</td>
<td>27232</td>
<td>1.80</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.771 ± 0.015(^a)</td>
<td>0.134 ± 0.021(^a)</td>
</tr>
<tr>
<td>RXTE (3–5 keV)</td>
<td>153.02 ± 0.79</td>
<td>123.9</td>
<td>27344</td>
<td>1.89</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.800 ± 0.012(^a)</td>
<td>0.115 ± 0.016(^a)</td>
</tr>
<tr>
<td>RXTE (5–12 keV)</td>
<td>153.10 ± 0.66</td>
<td>156.36</td>
<td>27345</td>
<td>1.91</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.182 ± 0.009(^a)</td>
<td>0.114 ± 0.013(^a)</td>
</tr>
<tr>
<td>Ryle (15 GHz)</td>
<td>150.69 ± 0.55</td>
<td>257.30</td>
<td>12493</td>
<td>2.14</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-4.454 ± 0.008(^a)</td>
<td>0.106 ± 0.012(^a)</td>
</tr>
<tr>
<td>GBI (8.30 GHz)</td>
<td>147.30 ± 1.03</td>
<td>44.1</td>
<td>1547</td>
<td>1.12</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-4.184 ± 0.008(^a)</td>
<td>0.124 ± 0.013(^a)</td>
</tr>
<tr>
<td>GBI (2.25 GHz)</td>
<td>150.92 ± 0.59</td>
<td>37.3</td>
<td>1547</td>
<td>1.18</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-4.247 ± 0.011(^a)</td>
<td>0.107 ± 0.014(^a)</td>
</tr>
<tr>
<td>CGRO (20–100 keV)</td>
<td>151.04 ± 0.32</td>
<td>109.1</td>
<td>2511</td>
<td>2.28</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.324 ± 0.005(^a)</td>
<td>0.081 ± 0.007(^a)</td>
</tr>
<tr>
<td>CGRO (100–300 keV)</td>
<td>152.37 ± 0.51</td>
<td>41.9</td>
<td>2511</td>
<td>3.05</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>1.843 ± 0.007(^a)</td>
<td>0.070 ± 0.008(^a)</td>
</tr>
<tr>
<td>Ginga (1–6 keV)</td>
<td>152.13 ± 1.55</td>
<td>11.3</td>
<td>337</td>
<td>1.22</td>
<td>7 \times 10^{-6}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.717 ± 0.017(^a)</td>
<td>0.109 ± 0.024(^a)</td>
</tr>
<tr>
<td>Ginga (6–20 keV)</td>
<td>151.01 ± 1.52</td>
<td>10.1</td>
<td>337</td>
<td>1.19</td>
<td>2 \times 10^{-5}</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-1.179 ± 0.015(^a)</td>
<td>0.109 ± 0.024(^a)</td>
</tr>
<tr>
<td>Ariel 5 (3–6 keV)</td>
<td>151.33 ± 0.79</td>
<td>24.8</td>
<td>1972</td>
<td>1.30</td>
<td>(&lt; 10^{-10})</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-1.053 ± 0.011(^a)</td>
<td>0.074 ± 0.015(^a)</td>
</tr>
<tr>
<td>Ryle (15 GHz)</td>
<td>192.29 ± 1.04</td>
<td>281.8</td>
<td>12493</td>
<td>2.12</td>
<td>(&lt; 10^{-10})</td>
<td>33.1</td>
<td>33.1</td>
<td>(&lt; 10^{-10})</td>
<td>-4.461 ± 0.011(^b)</td>
<td>0.107 ± 0.011(^b)</td>
</tr>
<tr>
<td>GBI (2.25 GHz)</td>
<td>185.47 ± 1.10</td>
<td>27.9</td>
<td>1547</td>
<td>1.19</td>
<td>(&lt; 10^{-10})</td>
<td>17.8</td>
<td>17.8</td>
<td>(&lt; 10^{-10})</td>
<td>-4.257 ± 0.017(^b)</td>
<td>0.052 ± 0.024(^b)</td>
</tr>
<tr>
<td>Ginga (1–6 keV)</td>
<td>230.30 ± 3.39</td>
<td>17.3</td>
<td>337</td>
<td>1.26</td>
<td>2 \times 10^{-8}</td>
<td>21.4</td>
<td>21.4</td>
<td>4 \times 10^{-7}</td>
<td>-0.701 ± 0.018(^c)</td>
<td>0.106 ± 0.026(^c)</td>
</tr>
<tr>
<td>Ginga (6–20 keV)</td>
<td>232.65 ± 3.35</td>
<td>11.8</td>
<td>337</td>
<td>1.11</td>
<td>1 \times 10^{-6}</td>
<td>20.9</td>
<td>20.9</td>
<td>2 \times 10^{-5}</td>
<td>-1.139 ± 0.014(^c)</td>
<td>0.042 ± 0.019(^e)</td>
</tr>
<tr>
<td>Ariel 5 (3–6 keV)</td>
<td>278.55 ± 3.07</td>
<td>81.7</td>
<td>1972</td>
<td>1.31</td>
<td>(&lt; 10^{-10})</td>
<td>18.6</td>
<td>18.6</td>
<td>(&lt; 10^{-10})</td>
<td>-1.074 ± 0.011(^c)</td>
<td>0.135 ± 0.015(^c)</td>
</tr>
<tr>
<td>Vela 5B (3–12 keV)</td>
<td>289.50 ± 1.70</td>
<td>19.3</td>
<td>1405</td>
<td>1.20</td>
<td>(&lt; 10^{-10})</td>
<td>42.8</td>
<td>42.8</td>
<td>1 \times 10^{-8}</td>
<td>4.721 ± 0.003(^c)</td>
<td>0.023 ± 0.004(^c)</td>
</tr>
</tbody>
</table>

\( ^a T_0 = \text{MJD 50514}.59, G_1 \) and \( \varphi_1 \) given for the common period of 151.43 d. \( ^b T_0 = \text{MJD 50514}.59, ^c T_0 = \text{MJD 44450}.00. \)
Figure 2. The spectral window of the RXTE/ASM light curve. The peaks at $2.2 \times 10^{-7}$ Hz (52.6 d) and $1.2 \times 10^{-5}$ Hz (0.96 d) appear to be caused by the precession of RXTE and observation scheduling, respectively.

Figure 3. Comparison of fits to the orbital modulation in the 1.5–3 keV range of the RXTE/ASM with one (dots) and three (solid curve) Fourier harmonics. We see that the single sinusoid fails to describe the shape of the modulation.

those of Paciesas et al. (1997), who found orbital modulation in the 20–300 keV BATSE data. The case for orbital modulation in the 100–300 keV band is still weaker. With the mhAoV method, we still find the period with a small error, $P = 5.597 \pm 0.005$, in spite of $\Theta = 1.0$. Overall, the BATSE data are fully compatible with the presence of (weak) modulation, but they do not prove it. On the other hand, an orbital modulation with parameters similar to those of our best fits is implied by the presence of Compton scattering in the companion wind, see Section 6.1.

The Ginga/ASM data show the orbital modulation at high significance (Table 2). Our results agree well with those of K00. We note here that the Ginga data clearly show higher depths of the modulation than the RXTE/ASM ones, especially for the 6–20 keV band. Taken at the face value, the data indicate that the wind from the companion was significantly stronger during 1987–1991 than that after 1996.

Surprisingly, we find no orbital modulation in the periodograms from Vela 5B/ASM and Ariel 5. This, however, agrees with the results of Holt et al. (1979) and PTH83, who reported that the 5.6-d modulation was not significantly detected in the Fourier transforms of those data.

Figure 4. The phase diagrams of the orbital modulation for the RXTE/ASM and radio data in the hard state. The solid curves represent the fits with $N = 3$. See Table 2 for details.

4.2 The soft state

We have analysed two occurrences of the soft state in the RXTE/ASM and Ryle data, 2001–2002 and 2003, see Fig. 1. The long duration of the former provides the best available constraints. The mhAoV periodograms for the RXTE/ASM data are shown in Fig. 5. We see no evidence for orbital modulation, confirming the result of W99 for the 1996 soft state. This is probably due to a strong increase of the degree of wind ionization by the enhanced soft X-ray flux (W99).

On the other hand, we find orbital modulation in the 15-GHz data in the soft state of 2001–2002, with $P = 5.6198 \pm 0.0043$ d at $P_1 = 0.012$ and $\phi_{mod} = (24.5 \pm 8.5)$ per cent. This modulation depth is compatible within errors with that in the hard state.
Figure 6. The low-frequency hard-state mHaOV periodograms (after removing the orbital modulation). (a) The RXTE/ASM data. (b) The CGRO/BATSE data. (c) The Ryle radio data. (d) The GBI radio data.

We find that each of the individual periods is consistent with the weighted average within $\lesssim 2\sigma$. Apart from the $P \sim 150$ d, we find also some longer periods, $>200$ d, in the earliest observations, i.e. by Vela 5B, Ariel 5, Ginga. For those, we have arbitrarily chosen $T_0 = 44450.0$ (MJD). We separately discuss the $\sim 290$-d period from the Vela 5B/ASM 10-yr monitoring in Appendix A.

Fig. 6(a) shows the hard-state RXTE/ASM periodograms. The most significant peak in all three energy bands corresponds to $P \simeq 150$ d. Similar values have been earlier reported in the 1.5–12 keV ASM data by B99, Benlloch et al. (2001, 2004) and Özdemir & Demircan (2001) and in optical data by Karitskaya et al. (2001). Also, that periodicity is confirmed by the SF analysis, see Appendix B. We find the fractional modulation in the three bands to be compatible with constant, see Table 3.

Pre-whitening with the $\sim 150$-d modulation has resulted in the lack of other significant periodicities in the RXTE/ASM data. Although we see two peaks with longer $P$ in the mHaOV periodograms, Fig. 6(a), their position change after pre-whitening with the orbital and the $\sim 150$-d periods. Also, those periods change when we consider shorter intervals of the ASM data. In particular, we do not confirm here the 182.5-d period claimed by Özdemir & Demircan (2001).

Fig. 6(b) shows the 20–100 and 100–300 keV mHaOV periodograms corresponding to the hard state observed by BATSE. The only significant peak corresponds to $P \sim 150$ d. It is also confirmed by the SF analysis, see Appendix B. Our findings contrast those of Paciesas et al. (1997), who reported no periodicities at frequencies $<0.1$ d$^{-1}$. Still, one can see a Fourier peak corresponding to $\sim 150$ d in their power spectrum. Then, B99 found $P \simeq 142.0 \pm 7.1$ d in the 20–100 keV data before the 1996 state transition, in agreement with our findings. Interestingly, the depth of the modulation is lower than that of the 1.5–12 keV emission, see Table 3.

The 15-GHz hard-state data show $P = 150.7 \pm 0.6$ d, see Fig. 6(c). However, that peak is accompanied by another statistically significant periodicity at 192.3 ± 1.0 d. We have checked that the $\sim 150$-d periodicity remains after pre-whitening with the $P \simeq 192$-d period. The 15-GHz data also show $P \simeq 293$-d period, similar to the $\sim 294$-d X-ray modulation found by PTH83 in the Vela 5B data. However, pre-whitening of the radio light curve with both the $\sim 150$- and 293-d periods causes the change of that longest period from 293 to $\sim 350$ d, which casts doubt on the reality of that modulation. In fact, neither the 192- nor 293-d periodicity appear applying the SF analysis.

The GBI 8.3- and 2.25-GHz data also show prominent modulation at $\sim 150$ d, see Table 3 and Figs 6(d) and 7. Their depth is compatible with being equal to that of the 15-GHz emission. Interestingly, the periodograms also show peaks around $\sim 190$ d, similar to those seen in the Ryle data. Subsets of the Ryle and GBI data up to 1998 were earlier analysed by B99, who obtained results compatible with ours.

The Ginga/ASM data were earlier analysed by K00. They reported two periods, 150 and $\sim 210$–230 d. Here, we confirm their results, obtaining two peaks at 151 and 230 d. In the Ariel 5 3–6 keV hard-state data, we have found two prominent peaks (not shown here). The first one at $P \simeq 278.6 \pm 3.1$ d is also confirmed by the SF analysis (Appendix B). It is close to the $P \simeq 296 \pm 10$ d reported by PTH83.

The second peak corresponds to $P \simeq 151.3 \pm 0.8$ d, which remains significant after pre-whitening with the first period. We note that the $\sim 150$-d periodicity is seen, in fact, in the Fourier spectrum of PTH83.

5 SUPERORBITAL MODULATION

Here, we search for variability with periods longer than the orbital one. We use the same method as in Section 4. However, we find that sufficient fits to the folded and averaged light curves can be obtained with $N = 1$. The main results are given in Table 3 and Figs 6 and 7.

We have found a $\sim 150$-d period to be the most pronounced superorbital one in the hard-state light curves. Note that we remove the orbital modulation, as described in Section 3.2 (except for Vela 5B/ASM and Ariel 5/ASM, which do not show orbital modulation), and other superorbital modulations (whenever present, see the bottom part of Table 3) prior to calculating the final results for the $\sim 150$-d superorbital modulation.

We then determine the common ephemeris to all the data using the weighted average, $151.43 \pm 0.20$ d, of all the $\sim 150$-d periods, and choose $T_0$ (around MJD 50500) based on the RXTE/ASM 5–12 keV data, which provide the highest signal-to-noise ratio of the modulation (see Table 3). This yields

$$\min[MJD] = 50514.59 + 151.43(\pm 0.20)E.$$ (12)
Figure 7. The phase diagrams of the superorbital modulation in the hard state for the ephemeris of equation (12), modelled by sinusoids. Note the approximate constancy of the phase for all the light curves. See Table 3 for the parameters.

Remarkably, all the ∼150-d modulations since 1976 are consistent with not only having the same period, but also show an almost constant phase, see Table 3 and Fig. 7. The phase constancy also supports our determination of the period from the weighted average, as an error on that would result in phase shifts in data separated by long intervals.

The approximately constant phase is kept in spite of a number of soft-state occurrences interrupting the hard state. The longest available soft-state period, of 2001–2002 is still too short (391 d, Table 1) to enable us to study the superorbital modulation in that state itself.

6 DISCUSSION

6.1 Orbital modulation

We consider the usual spherically symmetric approximation to the radial density profile of the wind from the companion,

\[ n(R) = \left( \frac{R_c}{R} \right)^2 \frac{n_0}{[1 - (R_c/R)]^\alpha} \]

(e.g. Castor, Abbott & Klein 1975), which corresponds to the velocity profile of

\[ v(R) = v_\infty \left( 1 - \frac{R_c}{R} \right)^\alpha, \]

and where \( R_c \) is the radius of the star, \( n_0 = \dot{M}_{\text{ion}}/(m_H 4\pi R_c^2 v_\infty) \) is the density parameter, \( \dot{M}_{\text{ion}} \) is the mass-loss rate, \( m_H \) is the hydrogen mass, \( v_\infty \) is the terminal velocity of the wind, and \( \alpha \) is the power-law index of the dependence.

These formulae were used by W99, who have successfully explained the orbital modulation seen in the hard state in their RXTE/ASM data by phase-dependent absorption in the ionized wind. They found \( i \simeq 30^{+10}_{-20} \) as the inclination implied by the modulation profile.

Here, we consider modulation due to Thomson scattering by the wind, relevant to the BATSE 20–100 keV energy range. We obtain then the fractional modulation depth (equation 7) of

\[ \phi_{\text{mod}} = \exp[-\tau_{\text{T}}(\pi)] - \exp[-\tau_{\text{T}}(0)] \]

\[ \exp[-\tau_{\text{T}}(\pi)], \] (15)

where \( \tau_{\text{T}} \) is the Thomson optical depth integrated along the line of
Then, and may be different from 3/7 (e.g. Larwood 1997). The Roche lobe radius is well approximated by,

$$R_l \approx \frac{0.49}{0.6 + \mu^{2/3} \ln(1 + \mu^{-1/3})}$$  

(Eggleton 1983). At $2 \leq \mu \leq 3$ estimated for Cyg X-1 (Gies & Bolton 1986; G03; Ziolkowski 2005), $R_l/a \approx 0.32–0.29$. Then,

$$R_l \approx 10^{12} \left( \frac{M_1 + M_2}{50 M_\odot} \right)^{1/3} \text{ cm.}$$  

Figure 8. The fractional orbital modulation due to Thomson scattering in the wind as the function of the inclination, see Section 6.1.

Figure 9. Schematic view of Cyg X-1 at the zero orbital phase. As before, $i$ is the orbital inclination, and $\delta$ is the angle between the inclined disc and the orbital plane.

6.2 Superorbital modulation

A generally accepted interpretation for the superorbital modulation, observed also in a number of other binary X-ray sources (e.g. SS 433, Her X-1, LMC X-3, LMC X-4), is accretion disc precession (see e.g. Wijers & Pringle 1999, for a review and references).

If, due to some reason, the disc is inclined with respect to the orbital plane (see Fig. 9), it will retrogradely precess due to the tidal forces exerted by the secondary (Katz 1973). This appears to explain the superorbital periodicities in Her X-1, SS 433 and LMC X-4 (Gerend & Boynton 1976; Leibowitz 1984; Heemskerk & van Paradijs 1989). The period of the tidally forced precession is given by,

$$\frac{P_{\text{orb}}}{P_{\text{sup}}} \approx \frac{3}{7} \left( \frac{\mu R_l}{a} \right)^{3/2} \frac{\mu}{(1 + \mu)^{1/2}} \cos \delta,$$  

where $\mu$ is the ratio of the mass of the secondary to that of the compact object, $R_l$ is the Roche lobe radius of the primary, $\beta$ is the ratio of the outer disc radius to $R_l$, and $\delta$ is the inclination of the disc with respect to the orbital plane (Larwood 1998). Equation (17) assumes that the disc precesses as a rigid body, which is the case when the sound crossing time in the disc is much less than the precession time-scale (Papaloizou & Terquem 1995). We note that the numerical coefficient above depends on the assumed disc model, and may be different from 3/7 (e.g. Larwood 1997).

The Roche lobe radius is well approximated by,

$$R_l \approx \frac{0.49}{0.6 + \mu^{2/3} \ln(1 + \mu^{-1/3})}$$  

(Eggleton 1983). At $2 \leq \mu \leq 3$ estimated for Cyg X-1 (Gies & Bolton 1986; G03; Ziolkowski 2005), $R_l/a \approx 0.32–0.29$. Then,

$$R_l \approx 10^{12} \left( \frac{M_1 + M_2}{50 M_\odot} \right)^{1/3} \text{ cm.}$$  

In the case of accretion via Roche lobe overflow and for thin and weakly viscous discs (Paczynski 1977), $\beta \approx 0.68–0.66$ at $2 \leq \mu \leq 3$. Then for viscous Roche lobe flows, the size of the disc corresponds to the tidal radius, for which $\beta \approx 0.87$ (Papaloizou & Pringle 1977). However, Cyg X-1 accretes via focused wind, in which case the disc size is usually smaller (e.g. Frank, King & Raine 2002). Therefore, equation (17) puts rather weak constraints on the amplitude of precession, $\delta$. At $\mu = 2$, $\delta \approx 0°–60°$ for $\beta \approx 0.55–0.87$, and at $\mu = 3$, $\delta \approx 0°–63°$ for $\beta \approx 0.52–0.87$. Still, equation (17) shows that the precession in Cyg X-1 is fully consistent with being driven by the tidal force of the companion.

On the other hand, Wijers & Pringle (1999) have considered precession due to radiation-induced warping in X-ray binaries. This can yield either retrograde or prograde precession. The corresponding precession period is roughly given by (see their equation 18).
where \( \dot{M} \) is the accretion rate, \( \alpha \) is the viscosity parameter and \( \epsilon \) is the accretion efficiency. For Cyg X-1,

\[
\mathcal{P}_{\text{warp}} \sim 2000 \left( \frac{\alpha}{0.1} \right)^{-4/5} \left( \frac{\epsilon}{0.1} \right)^{-1} \left( \frac{M}{10M_\odot} \right)^{3/4} \left( \frac{\dot{M}}{10^{17} \text{ g s}^{-1}} \right)^{-3/10} \left( \frac{\beta R_L}{10^{12} \text{ cm}} \right)^{3/4} \text{d},
\]

where \( M \) is the accretion rate, \( \alpha \) is the viscosity parameter and \( \epsilon \) is the accretion efficiency. For Cyg X-1,

\[
M \simeq 2 \times 10^{17} \frac{(F)}{4 \times 10^{-8} \text{ erg cm}^{-2} \text{ s}^{-1}} \left( \frac{d}{2 \text{kpc}} \right)^{-2} \left( \frac{\epsilon}{0.1} \right)^{-1} \text{ g s}^{-1},
\]

where \( (F) \) is the average bolometric hard-state flux and \( d \) is the distance, and we used above their values of Zdziarski et al. (2002) and Ziółkowski (2005), respectively.

The above estimates yield \( \mathcal{P}_{\text{warp}} \sim 10^4 \text{ d} \), much longer than that observed. We note that Wijers & Pringle (1999) have given an estimate of \( \mathcal{P}_{\text{warp}} \sim 180 \text{ d} \) for Cyg X-1, but assuming \( \alpha \) and \( M \) as high as 1 and \( 1.4 \times 10^{15} \text{ g s}^{-1} \), respectively. Thus, we consider radiation warping as an unlikely cause of the precession of Cyg X-1. Furthermore, the observed precession has a remarkably stable period, changing at most weakly over \( \gtrsim 15 \text{ yr} \), whereas Wijers & Pringle (1999) note that their mechanism is unlikely to give a stable period for sources with variable luminosity.

Regardless of the mechanism causing the precession, there is the issue of the mechanism causing the flux modulation during the precession. In principle, there are a number of possibilities. One is that the outer edge of the disc (completely optically thick) partially covers the X-ray source. This, however, would require extreme fine-tuning to achieve a \( \sim 25 \) per cent depth of the modulation. Namely, the X-ray source has the size \( \sim 10^2 R_g \) (where \( R_g \equiv GM/c^2 \)), as indicated by the X-ray power spectrum extending to high frequencies and agreement with theoretical prediction on the range of radii where most of the accretion power is released. On the other hand, the outer edge of the disc is at a much larger distance, \( \gtrsim 10^4 R_g \) (see the discussion above). Another possibility is that the outer part of the disc fully obscures the X-ray source, but we see X-rays scattered in a large-size corona above the disc. This, however, would dramatically affect the X-ray power spectrum, resulting in a cut-off above \( \sim 0.1 \text{ Hz} \), which is clearly not seen. Then, bound–free absorption in a spatially extended, moderately optically thin, medium associated with the outer regions of the disc appears to be ruled out as there is rather weak energy dependence of the modulation. On the other hand, a viable scenario is the outer disc wind/corona being almost fully ionized, with scattering away from the line of sight being responsible for the superorbital modulation.

Yet another possibility is that the X-ray emission is intrinsically anisotropic. Such a possibility was considered by B99, who relied in calculating the implied inclination on the blackbody-type anisotropy of a slab with the flux proportional to the projected area. However, there is overwhelming evidence that the dominant radiative process producing X-rays in the hard state of Cyg X-1 (and other black hole binaries) is thermal Comptonization (e.g. Zdziarski & Gierliński 2004). The anisotropy of thermal Comptonization (Poutanen & Svensson 1996) in planar geometries, though substantially weaker than that of the constant specific intensity case, appears fully capable to explain the observed superorbital modulation (work in preparation).

We note that our result on the stability of the superorbital period over \( \gtrsim 15 \text{ yr} \) rules out the model for strong outbursts of Cyg X-1 by Romero, Kaufman Bernado & Mirabel (2002). Those outbursts were observed (Stern, Beloborodov & Poutanen 2001; Golenetskii et al. 2002, 2003) on MJD 49727, 49801, 51287, 51289, 52329, 52364 and 52682. Romero et al. (2002) have proposed that the events are caused by precession of the jet of Cyg X-1, with the flares corresponding to the strongly beamed emission of the jet crossing our line of sight every precession period. They considered, in particular, the flares on MJD 49727, 49801 and 52329, reported by Golenetskii et al. (2002). The first two are separated by 74 d, which is about a half of the precession period of Cyg X-1. Romero et al. (2002) proposed that that precession period undergoes occasional changes, which would explain the value of 74 d. However, our present results show a remarkable stability of the precession period over the last several tens of years and rule out that explanation. Furthermore, the larger number of the dates of the outbursts compiled by Golenetskii et al. (2003) do not show any regularity. Interestingly, the event on MJD 52329 took place at the minimum of \( \sim 150 \text{ d} \) superorbital cycle.

With the RXTE/ASM data, we have also looked for signals corresponding to the beat between the orbital and superorbital frequencies. If the physical cause of the superorbital frequency is modulation of the X-ray flux emitted in a given direction by disc precession, the photons reflected (e.g. Magdziarz & Zdziarski 1995) from the surface of the secondary will change at the beat frequency of the two modulations. Then, the position of a possible peak in the periodogram at either \( v_{\text{orb}} + v_{\text{sup}} \) or \( v_{\text{orb}} - v_{\text{sup}} \) would tell us whether the precession is retrograde or prograde. We have thus searched for the corresponding peaks at the periods of 5.40 and 5.82 d.

Fig. 10(a) shows the periodograms for the 1.5–3 keV band before and after pre-whitening with both the orbital (calculated for

![Figure 10](image-url)
N = 3) and superorbital frequencies. After the pre-whitening, the use of N = 1 is sufficient to account for the shape of any remaining features. We clearly see a pronounced peak at 5.82 d. Fig. 10(b) shows that such peaks are present in all three ASM bands. The statistical significance of the features, $P_{\text{mod}}$, and the relative modulation depth, $\phi_{\text{mod}}$, for the 1.5–3, 3–5 and 5–12 keV bands is $1 \times 10^{-9}$ and 0.05, $2.2 \times 10^{-8}$ and 0.03, $2 \times 10^{-5}$ and 0.01, respectively. No statistically significant 5.40-d variability is detected. On the other hand, there is a rather strong peak (though much weaker than that at 5.82 d) at $\sim$5.5 d in the 1.5–3 keV band, which origin is unclear.

This may indicate that the precession in Cyg X-1 is prograde. However, the interpretation of the beat frequency as due to variable reflection from the companion surface presents some problems. Taking the determination of the binary parameters of Ziolkowski (2005), we find that the companion subtends a solid angle of $\sim 0.05 \times 4\pi$ as seen from the black hole. The observed modulation amplitude of $\sim 0.05$ in the 1.5–3 keV band would require a unit albedo and such precession amplitude that the X-ray source seen from the star surface is fully obscured by the disc. Furthermore, the albedo has to decrease by a factor of several in the 5–12 keV band. We have also checked the relative phase between the orbital and beat modulations. In the above interpretation, the two modulations should be in phase around the minimum of the superorbital modulation, when the disc is most inclined towards the companion. However, we find the ephemeris of the beat modulation to be

$$\text{min}[\text{MJD}] = 50239.70 + 5.82E,$$

which corresponds to the orbital and beat modulations being in phase instead around the midpoint between the minimum and the maximum of the superorbital modulation. Such relative phases appear to have no geometrical interpretation.

We have also searched for signatures of nutation. This effect is due to a perturbation of the disc rotation at the maximum of the torque exerted on the disc by the companion. This takes place when the star is closest to the point on the outer edge of the disc at the maximum distance from the orbital plane. Therefore, the nutation frequency is twice the beat frequency. We have not found signatures of nutation in any of our data.

We would like to stress the remarkable stability of the $\sim 150$-d modulation. This modulation is clearly present starting with the Ariel 5 data used by us, i.e. the beginning of 1976, and remains until at least mid 2003. This corresponds to about 65 precession periods. Furthermore, the phase of the modulation has also remained approximately constant, see Table 3. This appears to present a problem for the interpretation of the prograde character of precession. Precession can be prograde due to radiation-induced warping (Wijers & Pringle 1999), but it is then unlikely to have such constant period given the variable intrinsic luminosity of Cyg X-1 and its state transitions. Superhump-type precession is prograde, but it can take place only for $\mu \ll 1$ (e.g. Frank et al. 2002). Then, our results may indicate the presence of a new mechanism causing stable prograde precession at $\mu > 1$.

On the other hand, the $\sim 150$-d period is not present in the Vela 5b data, $\sim 1969$–1979, where, instead, the $\sim 290$-d period appears (Appendix A; PTH83). Interestingly, we may see a slow evolution of this period in a following time interval, with secondary superorbital periods in the Ariel 5 and Ginga data of 278 and 230 d, respectively. (The change of the long period from $\sim 290$ d to a half of this value was noticed by B99.)

## 7 CONCLUSIONS

We have studied periodic long-term variability of Cyg X-1 using a statistical method different from those used before, and apply it, for the first time, to the data set spanning over $\sim 30$ yr. In our method, we calculate the periodograms and probabilities with the mhAoV, and then fit the folded and averaged light curves. We have analysed X-ray monitoring data from Vela 5B, Ariel 5, Ginga, CGRO and RXTE, and radio data from the Ryle and Green Bank telescopes. We have confirmed and refined previous results on the 5.6-d orbital modulation of X-ray and radio emission in the hard state, caused by the attenuation in the wind of the companion supergiant being dependent the binary orbital phase. In particular, we show the detailed non-sinusoidal shape of the orbital modulation of X-rays observed by the RXTE/ASM and of the 15-GHz emission. For the first time, we find an orbital modulation at 15 GHz in the soft state. We confirm the presence of orbital modulation in the 20–100 keV CGRO/BATSE data.

Then, we find the presence of a $\sim 150$-d superperiodical period in all of the data since $\sim 1976$; in particular, we find its presence for the first time in the Ariel 5 data. Very remarkably, we find both that period and its phase have been stable over $> 65$ superorbital cycles. This coherence poses severe restrictions on any underlying clock mechanism. We find the cause of the superorbital modulation to be compatible with accretion disc precession, probably due to the tidal forces exerted by the secondary. Furthermore, we find modulation at the frequency corresponding to the beat between the orbital modulation and the prograde disc precession. On the other hand, we confirm the presence of a $\sim 290$-d periodicity in the 1969–1979 Vela 5B data. This indicates a switch from that precession period to its first harmonic around $\sim 1980$.

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The earliest reported long-term periodicity found in Cyg X-1 data is the $294 \pm 4$ d one found by *Vela 5B/ASM* in the $3-12$ keV range (PTH83). The same period was shortly afterward found in optical photometric data by Kemp et al. (1983). In Section 5, we have found a similar period of $279 \pm 3$ d in the $3-6$ keV *Ariel 5/ASM* data, as well as a tentative $\sim 293$-d period in the 15-GHz radio data. However, that period has not been confirmed in any other X-ray data.

Here, we analyse the *Vela 5B/ASM* observations of Cyg X-1 binned to $0.25$ d averages using the mhAoV method. The resulting periodogram is shown in Fig. A1. We find a very strong dominant peak at the period of $289.5 \pm 1.7$ d and the absence of any significant feature at the first harmonic. The significance, phase and the depth of the modulation are given in Table 3. We have also rebinned the data to $0.25$ d bins, as done by PTH83, and found the maximum slightly shifted to $294 \pm 4$ d. Thus, our results are in complete agreement with those of PTH83.

We should note that the precession period of *Vela 5B* is $\sim 300$ d, as seen in the *Vela 5B/ASM Calibration Guide of HEASARC*. This could, in principle, suggest an instrumental origin of the observed periodicity. In order to test this possibility, we have obtained *Vela 5B/ASM* periodograms for three other bright X-ray sources, 4U 0614+09, 4U 1940+09, and 3C 273, using the same time span (see Table 1), energy band and binning as for Cyg X-1. In all of those sources, we have found no significant periodicities around $\sim 300$ d. These results as well as the confirmation of the $\sim 290$-d period in the optical data (Kemp et al. 1983) appear to confirm the reality of the peak in the *Vela 5B/ASM* periodogram.

To further test the origin of the observed periodicity, we have generated a Monte Carlo light curve assuming the count rate equal $\sim 290$-d periodicity in the *Vela 5B/ASM* data.
to the average one of Cyg X-1 with a Gaussian distribution of the errors at the signal-to-noise ratio matching the data at the times matching those of the observations. We have then calculated the mhAoV periodogram and found no significant \( \sim 300\) d or other flux periodicity. In fact, \( \theta < 5 \) was found in the 0.003–0.2 \( d^{-1} \) frequency range. This further confirms the reality of the presence of the \( \sim 290\)-d period in the data.

We have also examined the periodograms for the time interval when Cyg X-1 was observed simultaneously by Ariel 5/ASM and Vela 5B/ASM, i.e. MJD 42830–44042. We have found those periodograms have the strongest peaks at the periods of 302 and 291 d, respectively. There is a \( \sim 150\)-d peak in the Ariel 5 data, but only as the third strongest one, and no corresponding peak is present in the Vela 5B data. This further supports our findings above.

It then appears that the superorbital period of Cyg X-1 had indeed changed over a time-scale \( \geq 10\) yr from \( \sim 290\)-d to approximately its first harmonic, \( \sim 150\) d, seen in all the observations after those by Vela 5B. During a transitional time interval, both periods were present, see Table 3.

### APPENDIX B: THE STRUCTURE–FUNCTION OF CYG X-1

A method to quantify long-term variability alternative to periodograms is to calculate the SF. It was introduced by Kolomogorov (1941a,b), and applied for the first time in astronomy by Simonetti, Cordes & Heeschen (1985), and then used by many authors, e.g. Hughes, Aller & Aller (1992), Collier & Peterson (2001) and Czerny et al. (2003).

It is a function of the time domain, with the first-order SF defined by

\[
\text{SF}(\tau) = \langle |F(t + \tau) - F(t)|^2 \rangle,
\]

where \( F(t) \) is the light curve. If \( F(t) = A + \sin t \), i.e. it is sinusoidal with the \( 2\pi \) period and we measure it between the times \( a \) and \( b \), the SF can be calculated to be,

\[
\text{SF}(\tau) = \frac{1}{b-a} \int_a^{b-\tau} [\sin(t + \tau) - \sin t]^2 \, dt
\]

\[
= 2\sin^2 \frac{\tau}{2} \left[ 1 - \frac{2 \cos(a + \tau) \sin(a - \tau)}{b - a - \tau} \right],
\]

where \( \tau < b - a \), and the second (boundary) term in brackets is negligible for \( \tau \ll b - a \). We see that the SF has minima (at 0) for \( \tau \) equal to the period and its subharmonics, i.e. \( \tau = 2k\pi \), where \( k \) is an integer. This should be bear in mind when analysing results of application of this technique to observed light curves.

We then search for long-term periodicities in our light curves corresponding to the hard state (except that of Ginga, due to its limited quality). We use the procedure described in Czerny et al. (2003) for calculating the SF and its uncertainty. The results are shown in Fig. B1.

We clearly see the dips to the \( \sim 150\)-d period and its subharmonics in the RXTE/ASM, BATSE and Ryle data. In the ASM data, the dip position is at 158 \( \pm 14\) d. (Hereafter, the uncertainty has been estimated as equal to the full width at half-maximum of the absolute value of the dip profile after subtracting the surrounding continuum.) Thus, this result is consistent with our periodogram results. In the 20–100 keV BATSE data, the dip is at 153 \( \pm 10\) d, while the 100–300 keV band shows only a shallow minimum at \( \sim 150\) d. The radio data show the dip at 151 \( \pm 19\) d. Interestingly, there is no sign of any dip at \( \sim 192\) d, which was found in the corresponding periodogram (see Section 5). On the other hand, the Ariel 5/ASM show a pronounced dip at \( \sim 276\) d. The Vela 5B show only a very shallow minimum around \( \sim 300\) d.

Thus, the SF technique fully confirms the periodogram results for the RXTE/ASM and for the 20–100 keV BATSE band. Thus, we can consider those results as beyond any reasonable doubt. For other data, the SF results are in only partial agreement with the periodogram ones.

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